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Wavelet Transform of Even- and Odd-Coherent States

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Abstract By employing the technique of integral within an ordered product (IWOP) of operators we recast classical wavelet transform to a matrix element of the squeezing-displacing operator $U(\mu, s)$ between the mother wavelet vector $\langle\psi|$ and the state vector $|f\rangle$ to be transformed, i.e., we propose that $\langle\psi|U(\mu, s)|f\rangle$ can be considered as a new kind of spectrum for analyzing the quantum state $|f\rangle$. In this way we do numerical calculation of wavelet-transform spectrum for the even- and odd-coherent states and then plot their figures, respectively. Thus this kind of spectrum can be used to recognize a variety of quantum optical states.

Keywords Wavelet transform · The even- and odd-coherent states · The IWOP technique

1 Introduction

Wavelet theory had its origin in quantum field theory, signal analysis, and function space theory. In these areas wavelet-like algorithms replace the classical Fourier-type expansion of a function, since it can overcome some shortcomings of classical Fourier analysis [1–7]. In this work we shall employ the newly developed technique of integration within an ordered product (IWOP) of operators [8, 9] to recast the classical wavelet transform to a matrix element of the squeezing-displacing operator $U(\mu, s)$ between the mother wavelet vector $\langle\psi|$ and the state vector $|f\rangle$ to be transformed, i.e., we propose that $\langle\psi|U(\mu, s)|f\rangle$ can be considered as a new kind of spectrum for analyzing the quantum state $|f\rangle$. In this paper we shall study the spectrum for the even- and odd-coherent states. We do numerical calculation of wavelet-transform spectrum for the even- and odd-coherent states and then plot their

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figures, respectively. In these figures all even-coherent states are even-symmetric with respect to $s = 0$ plane, whereas all odd-coherent states are odd-symmetric. Thus this kind of spectrum can be used to recognize a variety of quantum optical states.

2 Wavelet Transform of Quantum States

Wavelets are defined by starting with a function of the real variable x, named a mother wavelet if it is well localized and oscillating. (It is called wavelet because it is localized and it resembles a wave because it oscillates.) The localization condition means that the wave $\psi(x)$ decreases rapidly to zero as $|x|$ tends to infinity. Mathematically it is required that the integral of $\psi(x)$ be zero

$$\int_{-\infty}^{\infty} \psi(x)dx = 0. \tag{1}$$

A more general requirement for a mother wavelet is to demanded $\psi(x)$ to have vanishing moments $\int_{-\infty}^{\infty} x^l \psi(x)dx = 0$, $l = 0, 1, 2, \ldots, L$. (A greater degree of smoothness than continuity also leads to vanishing moments for the mother wavelet.) The theory of wavelets is concerned with the representation of a function in terms of a two-parameter family of dilates and translates of a fixed function.

The mother wavelet $\psi(x)$ generates the other wavelets of the family $\psi_{(\mu,s)}$, (μ is named scaling parameter, $\mu > 0$, s is a translation parameter, $s \in R$), the dilated-translated function is defined as

$$\psi_{(\mu,s)} = \frac{1}{\sqrt{\mu}} \psi\left(\frac{x-s}{\mu}\right). \tag{2}$$

For quantum mechanics we are concerned with wave function $f(x)$ analysis and its evolution. The continuous wavelet transform of a wave function $f(x) \in L^2(R)$ with respect to ψ is defined by the integral

$$Wf(\mu, s) = \frac{1}{\sqrt{\mu}} \int_{-\infty}^{\infty} f(x)\psi^*\left(\frac{x-s}{\mu}\right)dx. \tag{3}$$

In [10], based on Dirac's symbolic method, Fan and Lu have expressed (3) as a matrix element of the squeezing-translating operator $U(\mu, s)$

$$Wf(\mu, s) = \langle\psi|U(\mu, s)|f\rangle, \tag{4}$$

where $\langle\psi|$ is the state vector corresponding to the given mother wavelet, $|f\rangle$ is a quantum state to be transformed, and

$$U(\mu, s) \equiv \frac{1}{\sqrt{\mu}} \int_{-\infty}^{\infty} \left|\frac{x-s}{\mu}\right\rangle\langle x|dx \tag{5}$$

is the squeezing-translating operator, $|x\rangle$ is the eigenvector of coordinate X, $X|x\rangle = x|x\rangle$.

In order to combine the wavelet transform with quantum states transform more tightly, we recall that in Fock space, $|x\rangle$ is expressed as ($X = (a + a^\dagger)/\sqrt{2}$) [11]

$$|x\rangle = \pi^{-\frac{1}{4}} \exp\left(-\frac{1}{2}x^2 + \sqrt{2}xa^\dagger - \frac{a^{\dagger 2}}{2}\right)|0\rangle, \tag{6}$$

where $|0\rangle$ is the vacuum state annihilated by the Bose annihilation operator a with $[a, a^\dagger] = 1$. Putting (6) into (5) and using the normal product form of the vacuum projector

$$|0\rangle\langle 0| = :\exp(-a^\dagger a):, \tag{7}$$

here :: denotes the normal ordering, as well as the IWOP technique, we can directly perform the integration in (5) with the result [12, 13]

$$\begin{aligned} U(\mu, s) = A \exp\left(-\frac{a^{\dagger 2}}{2}\tanh\lambda - \frac{a^\dagger s}{\sqrt{2}}\operatorname{sech}\lambda\right):\exp[(\operatorname{sech}\lambda - 1)a^\dagger a]: \\ \times \exp\left(\frac{a^2}{2}\tanh\lambda + \frac{as}{\sqrt{2}\mu}\operatorname{sech}\lambda\right), \end{aligned} \tag{8}$$

where $A \equiv \operatorname{sech}^{\frac{1}{2}}\lambda \exp[-\frac{s^2}{2(\mu^2+1)}]$ and

$$\operatorname{sech}\lambda = \frac{2\mu}{\mu^2+1}, \qquad \tanh\lambda = \frac{\mu^2-1}{\mu^2+1}, \quad \mu = e^\lambda. \tag{9}$$

Equation (8) is the explicitly normal product form of $U(\mu, s)$.

Further, it follows from the following operator identity

$$e^{\lambda a^\dagger a} = :\exp\left[(e^\lambda - 1)a^\dagger a\right]: \tag{10}$$

that

$$\begin{aligned} U(\mu, s) = A \exp\left[-\frac{a^{\dagger 2}}{2}\tanh\lambda - \frac{a^\dagger s}{\sqrt{2}}\operatorname{sech}\lambda\right] \\ \times \exp[a^\dagger a \ln\operatorname{sech}\lambda] \exp\left[\frac{a^2}{2}\tanh\lambda + \frac{as}{\sqrt{2}\mu}\operatorname{sech}\lambda\right]. \end{aligned} \tag{11}$$

In particular, when $s = 0$, it reduces to the well-known squeezing operator,

$$U(\mu, 0) = \frac{1}{\sqrt{\mu}}\int_{-\infty}^{\infty}\left|\frac{x}{\mu}\right\rangle\langle x| dx = \exp\left[\frac{\lambda}{2}\left(a^2 - a^{\dagger 2}\right)\right], \tag{12}$$

when $\mu = 1$, to the displace operator,

$$U(0, s) = \int_{-\infty}^{\infty}|x - s\rangle\langle x| dx = \exp\left[\frac{s}{\sqrt{2}}\left(a - a^\dagger\right)\right]. \tag{13}$$

From (4) and (5), once the state vector $\langle\psi|$ corresponding to mother wavelet is known, for any quantum state $|f\rangle$ the matrix element $\langle\psi|U(\mu, s)|f\rangle$ is just the wavelet transform of wave function $f(x)$ with respect to $\langle\psi|$, thus the wavelet transform of $|f\rangle$ with different parameters (μ, s) can be used to describe quantum state $|f\rangle$ in a some manner. Therefore, this kind of spectrum may play the role in helping us to recognize the quantum states.

3 General Formula for Finding Mother Wavelets

Now we analyze the condition (1) for mother wavelet from the point of view of quantum mechanics. Because

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}|x\rangle dx=|p=0\rangle, \tag{14}$$

where $|p\rangle$ is the momentum eigenvector, $|p\rangle=\pi^{-\frac{1}{4}}\exp(-\frac{1}{2}p^2+i\sqrt{2}pa^\dagger+\frac{a^{\dagger 2}}{2})|0\rangle$, we can recast the condition (1) into Dirac's bra-ket formalism

$$\int_{-\infty}^{\infty}\psi(x)dx=0 \quad\to\quad \langle p=0|\psi\rangle=0. \tag{15}$$

We want to find a value of $|\psi\rangle$ that obeys (15). By considering that $|n\rangle=\frac{a^{\dagger n}}{\sqrt{n!}}|0\rangle$ is the basis in Fock space, without loss of generality, we suppose

$$|\psi\rangle=\sum_{n=0}^{\infty}g_n a^{\dagger n}|0\rangle, \tag{16}$$

where g_n are such that $|\psi\rangle$ obeys condition (15). Then, using the overcompleteness relation of coherent states, $|\alpha\rangle=\exp(\alpha a^\dagger-\alpha^* a)|0\rangle$, and $\langle p=0|\alpha\rangle=\pi^{-1/4}\exp(-|\alpha|^2/2+\alpha^2/2)$, we have

$$\begin{aligned}\langle p=0|\psi\rangle&=\langle p=0|\int\frac{d^2\alpha}{\pi}|\alpha\rangle\langle\alpha|\sum_{n=0}^{\infty}g_n a^{\dagger n}|0\rangle\\&=\pi^{-1/4}\sum_{n=0}^{\infty}g_n\int\frac{d^2\alpha}{\pi}e^{-|\alpha|^2}\alpha^{*n}\sum_m\frac{1}{m!}\left(\frac{\alpha^2}{2}\right)^m\\&=\pi^{-1/4}\sum_n\frac{(2n)!}{n!2^n}g_{2n}=0.\end{aligned} \tag{17}$$

Equation (17) provides a general formalism to find the qualified wavelets, e.g., assuming $g_{2n}=0$ for $n>2$, so the coefficients of the surviving terms should satisfy

$$g_0+g_2+3g_4=0 \tag{18}$$

and $|\psi\rangle$ becomes

$$|\psi\rangle=\left(g_0+g_2a^{\dagger 2}+g_4a^{\dagger 4}\right)|0\rangle. \tag{19}$$

Projecting (19) onto the coordinate representation, we get the qualified wavelets

$$\psi(x)=\pi^{-\frac{1}{4}}e^{-\frac{x^2}{2}}\left[g_0+g_2\left(2x^2-1\right)+g_4\left(4x^4-12x^2+3\right)\right], \tag{20}$$

where we used $\langle x|n\rangle=\frac{1}{\sqrt{2^n n!\sqrt{\pi}}}e^{-\frac{x^2}{2}}H_n(x)$ and $H_n(x)$ is the Hermite polynomials. When $g_0=-g_2=\frac{1}{2}$, and $g_4=0$, $\psi(x)\to\psi_1(x)$ in (20), which is the Mexican hat function $\psi_1(x)=\pi^{-1/4}e^{-x^2/2}(1-x^2)$ (see Fig. 1)

$$\begin{aligned}|\psi\rangle_1&=\int_{-\infty}^{\infty}dx|x\rangle\langle x|\psi\rangle_1=\pi^{-\frac{1}{4}}\int_{-\infty}^{\infty}dx(1-x^2)\exp\left(-x^2+\sqrt{2}xa^\dagger-\frac{a^{\dagger 2}}{2}\right)|0\rangle\\&=\frac{1}{2}\pi^{\frac{1}{4}}(1-a^{\dagger 2})|0\rangle.\end{aligned} \tag{21}$$

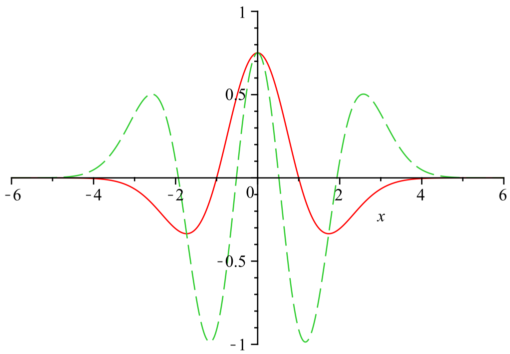


Fig. 1 Mathor wavelets (*solid curve shows* the Mexican hat wavelet $\psi_1(x)$, the *points* show the $\psi_2(x)$)

$|\psi\rangle_1$ is the statevector corresponding to it. When $g_0 = -g_4 = -\frac{1}{4}$, and $g_2 = -\frac{1}{2}$, from (20) we obtain the mother wavelet function (see Fig. 1)

$$\psi_2(x) = \pi^{-\frac{1}{4}} e^{-\frac{x^2}{2}} \left(1 - 4x^2 + x^4\right), \tag{22}$$

and the corresponding statevector

$$|\psi\rangle_2 = \frac{1}{4}\pi^{\frac{1}{4}} \left(-1 - 2a^{\dagger 2} + a^{\dagger 4}\right)|0\rangle. \tag{23}$$

Therefore, as long as parameters g_n conform to condition (17), we can adjust their values to control the shape of the wavelets.

4 Wavelet-Transform Spectrum of the Even and Odd Coherent States

Recently, in the field of quantum optics, much attention has been paid to superposition of quantum states (so-called quantum state engineering) because they exhibit nonclassical properties, such as sub-Poissonian statistics, antibunching, and quadrature squeezing [14, 15]. These states are closely related to some probability distribution functions. For examples, the even and odd coherent states $|\pm\alpha\rangle$, which can be given in the form

$$|\pm\alpha\rangle = N_\pm\big(|\alpha\rangle \pm |-\alpha\rangle\big), \tag{24}$$

where $|\alpha\rangle$ is a coherent state and $N_\pm$ are normalization constants

$$N_\pm = \frac{1}{\sqrt{2}}\left[1 \pm \exp\left(-2|\alpha|^2\right)\right]. \tag{25}$$

They are representative examples for the superposition of two coherent states, and can be interpreted as quantum superpositions of two macroscopically distinguishable states for a large amplitude α, so-called Schröinger cat states. It is worth noting that recently electronic states of Schröinger cat type have been prepared via pulsed excitation of atomic Rydberg wave packets.

As a typical example of superposition of quantum states, we calculate the wavelet transform of the even and odd coherent states to obtain its spectrum for the different parameters, and then analyze the results. We first calculate the matrix element $\langle n|U(\mu, s)|\alpha\rangle$, where $|n\rangle$ is the number state in Fock space.

It is well known that for a coherent state $|\alpha\rangle$, due to $a|\alpha\rangle = \alpha|\alpha\rangle$, matrix elements of any normally ordered $:F(a^\dagger, a):$ in the coherent state is easily obtained, i.e.,

$$\langle\beta|{:}F(a^\dagger, a){:}|\alpha\rangle = F(\beta^*, \alpha)\langle\beta|\alpha\rangle, \tag{26}$$

where $F(\beta^*, \alpha)$ is a function in phase space obtained by replacing $a^\dagger, a$ with β^*, α, respectively. Note that (26) is valid for the un-normalized coherent state as well. Using (8) and (26) and the un-normalized coherent state $\|\alpha\rangle = \exp(\alpha a^\dagger)|0\rangle$, $|\alpha\rangle = e^{-|\alpha|^2/2}\|\alpha\rangle$, leading to

$$|n\rangle = \frac{1}{\sqrt{n!}}\frac{d^n}{d\alpha^n}\|\alpha\rangle\bigg|_{\alpha=0}, \tag{27}$$

and

$$\langle\beta\|\alpha\rangle = \exp(\beta^*\alpha), \tag{28}$$

it is easy to see that the matrix element $\langle m|U(\mu, s)\|\alpha\rangle$ is given by

$$\begin{aligned}\langle n|U(\mu, s)\|\alpha\rangle &= \frac{1}{\sqrt{n!}}\frac{\partial^n}{\partial\beta^{*n}}\langle\beta\|U(\mu, s)\|\alpha\rangle\bigg|_{\beta^*=0}\\ &= \frac{A}{\sqrt{n!}}\frac{\partial^n}{\partial\beta^{*n}}\exp\bigg(-\frac{\tanh\lambda}{2}\beta^{*2} - \frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}}\beta^*\\ &\quad + \beta^*\alpha\,\mathrm{sech}\,\lambda + \frac{\tanh\lambda}{2}\alpha^2 + \frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}\mu}\alpha\bigg)\bigg|_{\beta^*=0}.\end{aligned} \tag{29}$$

Especially, when $n = 0$, $n = 2$ and $n = 4$, we have

$$\langle 0|U(\mu, s)\|\alpha\rangle = A\exp\left(\frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}\mu}\alpha + \frac{\tanh\lambda}{2}\alpha^2\right), \tag{30}$$

$$\begin{aligned}\langle 2|U(\mu, s)\|\alpha\rangle &= \frac{A}{\sqrt{2}}\left\{\left[-\tanh\lambda + \left(\alpha - \frac{s}{\sqrt{2}}\right)^2\mathrm{sech}^2\,\lambda\right]\right.\\ &\quad\left.\times\exp\left(\frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}\mu}\alpha + \frac{\tanh\lambda}{2}\alpha^2\right)\right\},\end{aligned} \tag{31}$$

and

$$\begin{aligned}&\langle 4|U(\mu, s)\|\alpha\rangle\\ &\quad= \frac{A}{\sqrt{24}}\left\{\left[3\tanh^2\lambda + 6\left(\alpha - \frac{s}{\sqrt{2}}\right)^2\tanh\lambda\,\mathrm{sech}^2\,\lambda + \left(\alpha - \frac{s}{\sqrt{2}}\right)^4\mathrm{sech}^4\,\lambda\right]\right.\\ &\quad\left.\times\exp\left(\frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}\mu}\alpha + \frac{\tanh\lambda}{2}\alpha^2\right)\right\}.\end{aligned} \tag{32}$$

Using (29) and considering (24), it is obtained from (30)–(32) that the wavelet-transform spectrum of even and odd coherent states $|\pm\alpha\rangle$ when the mother wavelet is a Mexican hat

$\psi_1(x)$ or the mother wavelet $\psi_2(x)$,

$$\begin{aligned}
{}_1\langle\psi|U(\mu,s)|\pm\alpha\rangle &= \frac{\pi^{\frac{1}{4}}N_\pm}{2}(\langle 0| - \sqrt{2}\langle 2|)U(\mu,s)(|\alpha\rangle \pm |-\alpha\rangle) \\
&= \frac{\pi^{\frac{1}{4}}N_\pm}{2}e^{-\frac{|\alpha|^2}{2}}\{\langle 0|U(\mu,s)\|\alpha\rangle \pm \langle 0|U(\mu,s)\|-\alpha\rangle \\
&\quad - \sqrt{2}[\langle 2|U(\mu,s)\|\alpha\rangle \pm \langle 2|U(\mu,s)\|-\alpha\rangle]\} \\
&= \frac{\pi^{\frac{1}{4}}AN_\pm}{2}e^{-\frac{|\alpha|^2}{2}+\frac{\tanh\lambda}{2}\alpha^2}\Bigg\{(1+\tanh\lambda)\big(e^{\frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}\mu}\alpha} \pm e^{-\frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}\mu}\alpha}\big) \\
&\quad - \mathrm{sech}^2\lambda\Bigg[\bigg(\alpha - \frac{s}{\sqrt{2}}\bigg)^2 e^{\frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}\mu}\alpha} \pm \bigg(\alpha + \frac{s}{\sqrt{2}}\bigg)^2 e^{-\frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}\mu}\alpha}\Bigg]\Bigg\}, \qquad (33)
\end{aligned}$$

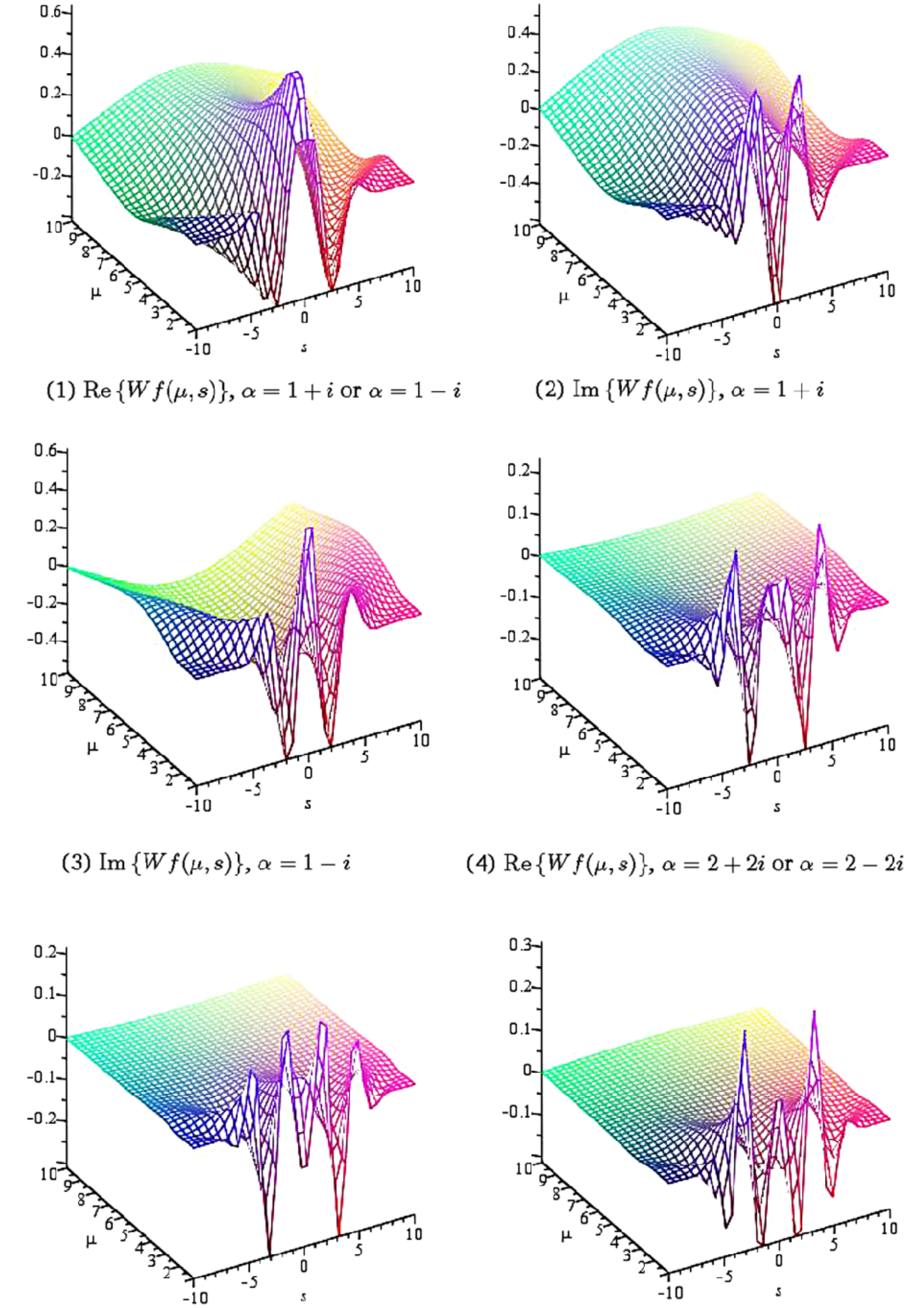
and

$$\begin{aligned}
&{}_2\langle\psi|U(\mu,s)|\pm\alpha\rangle \\
&= \frac{\pi^{\frac{1}{4}}}{4}N_\pm(-\langle 0| - 2\sqrt{2}\langle 2| + \sqrt{24}\langle 4|)U(\mu,s)(|\alpha\rangle \pm |-\alpha\rangle) \\
&= \frac{\pi^{\frac{1}{4}}AN_\pm}{4}e^{-\frac{|\alpha|^2}{2}+\frac{\tanh\lambda}{2}\alpha^2}\Bigg\{(-1+2\tanh\lambda+3\tanh^2\lambda)\big(e^{\frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}\mu}\alpha} \pm e^{-\frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}\mu}\alpha}\big) \\
&\quad + 2(3\tanh\lambda - 1)\,\mathrm{sech}^2\lambda\Bigg[\bigg(\alpha - \frac{s}{\sqrt{2}}\bigg)^2 e^{\frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}\mu}\alpha} \pm \bigg(\alpha + \frac{s}{\sqrt{2}}\bigg)^2 e^{-\frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}\mu}\alpha}\Bigg] \\
&\quad + \mathrm{sech}^4\lambda\Bigg[\bigg(\alpha - \frac{s}{\sqrt{2}}\bigg)^4 e^{\frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}\mu}\alpha} \pm \bigg(\alpha + \frac{s}{\sqrt{2}}\bigg)^4 e^{-\frac{s\,\mathrm{sech}\,\lambda}{\sqrt{2}\mu}\alpha}\Bigg]\Bigg\}. \qquad (34)
\end{aligned}$$

In the same way, we can obtain the wavelet-transform spectrum of even and odd coherent states $|\pm\alpha\rangle$ when the mother wavelets is any mother wavelets that conform to condition (1).

5 Numerical Results and Discussion

For visualizing the wavelet-transform spectra of the quantum states, the resulting value $\langle\psi|U(\mu,s)|f\rangle$ versus (μ,s) can be numerical calculated and plotted, with the two horizontal axes denoting scaling parameter μ and translation parameter s. Figures 2 and 3 are the wavelet-transform spectra of the even- and odd-coherent states when the mother wavelet is a Mexican hat $\psi_1(x)$, respectively. Because ${}_1\langle\psi|U(\mu,s)|\pm\alpha\rangle$ is complex, there are two figures corresponding to the real and imaginary parts when α take different values. These figures exhibit the common behavior that all peaks are narrow and local, which are characteristics of the wavelet transformation. We can see that the location and shape of the peaks change with different α values. In these figures, it is worth noting that all even-coherent states are even-symmetric with respect to $s=0$ plane, whereas all odd-coherent states are odd-symmetric. Thus, these results imply that this kind of spectrum can be used to recognize a variety of quantum optical states, and may have some potential applications in quantum information. Moreover, by filtering and anti-conversion for the wavelet transform spectrum, one may obtain other information which may represent new quantum optics states beneficial to quantum state engineering [16, 17].



(1) $\mathrm{Re}\{Wf(\mu,s)\}$, $\alpha=1+i$ or $\alpha=1-i$

(2) $\mathrm{Im}\{Wf(\mu,s)\}$, $\alpha=1+i$

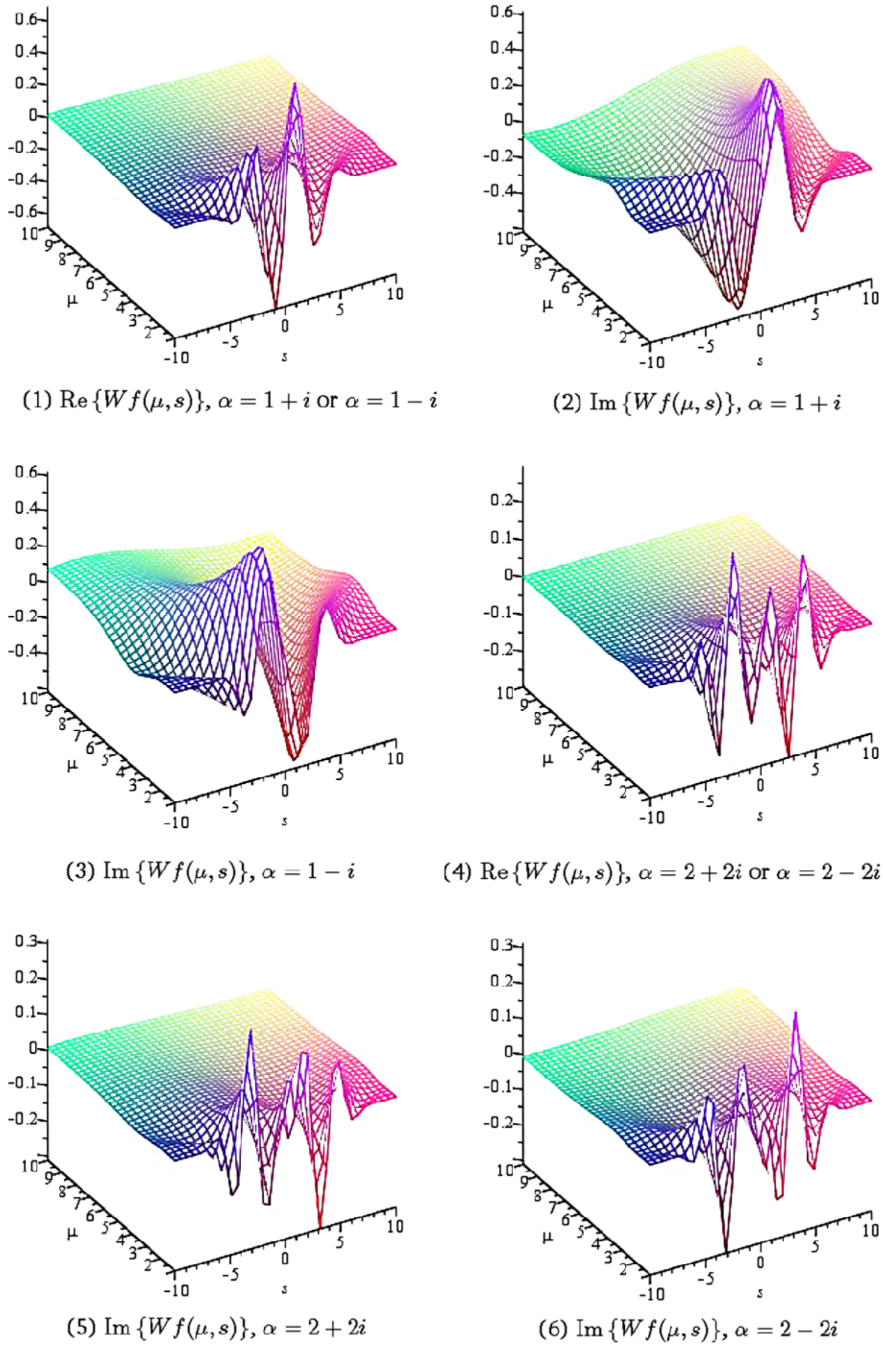
(3) $\mathrm{Im}\{Wf(\mu,s)\}$, $\alpha=1-i$

(4) $\mathrm{Re}\{Wf(\mu,s)\}$, $\alpha=2+2i$ or $\alpha=2-2i$

(5) $\mathrm{Im}\{Wf(\mu,s)\}$, $\alpha=2+2i$

(6) $\mathrm{Im}\{Wf(\mu,s)\}$, $\alpha=2-2i$

Fig. 2 Wavelets-transform spectra of the even coherent states by the Mexican hat wavelet $\psi_1(x) = \pi^{-1/4}e^{-x^2/2}(1-x^2)$



(1) $\mathrm{Re}\{Wf(\mu,s)\}$, $\alpha = 1+i$ or $\alpha = 1-i$

(2) $\mathrm{Im}\{Wf(\mu,s)\}$, $\alpha = 1+i$

(3) $\mathrm{Im}\{Wf(\mu,s)\}$, $\alpha = 1-i$

(4) $\mathrm{Re}\{Wf(\mu,s)\}$, $\alpha = 2+2i$ or $\alpha = 2-2i$

(5) $\mathrm{Im}\{Wf(\mu,s)\}$, $\alpha = 2+2i$

(6) $\mathrm{Im}\{Wf(\mu,s)\}$, $\alpha = 2-2i$

Fig. 3 Wavelets-transform spectra of the odd coherent states by the Mexican hat wavelet $\psi_1(x) = \pi^{-1/4}e^{-x^2/2}(1-x^2)$

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